those reported by Traube (*13*) shows that the latter's data are an average of 0.6% higher than those reported here. Two factors may contribute to this difference. Probably the concentrations Traube reports are slightly in error, because of his failure to use anhydrous acetone. The density of the material he started with is not reported and thus the water content cannot be estimated by the method described previously (*11*). The presence of only a trace quantity of water in the starting acetone would make his reported compositions high. Correction of this effect would bring his data into better agreement with those reported here. Considering the refinements in technique which have occurred since 1891, the agreement of the Traube data is excellent.

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LITERATURE CITED

1. Brunjes, A. S., and M. J. P. Bogart, *Ind. Eng. Chem.*, **35**, 258 (1948).
2. Harkins, W. D., "Technique of Organic Chemistry," (A. Weissberger, ed.), vol. I, Part 1, 2 ed., p. 366, Interscience Publishers, Inc., New York (1949).
3. *Ibid.*, p. 368.
4. *Ibid.*, p. 374.
5. (a) Harkins, W. D., "Physical Chemistry of Surface Films," p. 79, Reinhold Publishing Corporation, New York (1949).
   (b) Reference *2*, p. 369.
6. ———, and F. E. Brown, *J. Am. Chem. Soc.*, **41**, 499 (1919).
7. Harkins, H. N., and W. D. Harkins, *J. Clin. Investigations*, **7**, 263 (1929).
8. Morgan, J. L. R., and A. J. Scarlett, Jr., *J. Am. Chem. Soc.*, **39**, 2275 (1917).
9. Perry, J. H., "Chemical Engineers' Handbook," 3 ed., p. 175, McGraw-Hill Book Company, Inc., New York (1950).
10. Richards, T. W., and E. K. Carver, *J. Am. Chem. Soc.*, **43**, 827 (1921).
11. Thomas, K. T., and R. A. McAllister, *A.I.Ch.E. Journal*, **3**, 161 (1947).
12. Timmermans, J., "Physico-Chemical Constants of Pure Organic Compounds," p. 147, Elsevier Publishing Company, New York (1950).
13. Traube, J., *Ann. Chem. Justus Liebigs*, **265**, 27 (1891).
14. Young, T. F., and W. D. Harkins, in "International Critical Tables," vol. IV, p. 467, McGraw-Hill Book Company, Inc., New York (1928).

Simple Theory of an Idealized Turbulent Mixer

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With stationary isotropic turbulence postulated, the rate of decrease in concentration fluctuations of a scalar contaminant is estimated in terms of the turbulence scale and the power input to the system.

One of the simplest statistical measures of nonuniformity in a fluid mixer is the mean-square fluctuation in concentration.

A differential equation for $\overline{\gamma^2}(t)$ of a homogeneous field (i.e., all statistical functions invariant to translation) is obtained by multiplying the material conservation equation

$$\frac{\partial\gamma}{\partial t} + u_k\frac{\partial\gamma}{\partial x_k} = D\nabla^2\gamma \qquad (1)$$

(where u_k is turbulent velocity, and D is diffusivity by γ) and averaging. With homogeneity, this yields (*1*)

$$\frac{d\overline{\gamma^2}}{dt} = -2D\overline{\left(\frac{\partial\gamma}{\partial x_i}\right)\left(\frac{\partial\gamma}{\partial x_i}\right)} \qquad (2)$$

If restricted further to isotropic fields (i.e., all statistical functions invariant to rotation), the three mean-square derivatives can be characterized by a single length, analogous to G. I. Taylor's "microscale" of turbulence:

$$\overline{\left(\frac{\partial\gamma}{\partial x_i}\right)\left(\frac{\partial\gamma}{\partial x_i}\right)} = 6\,\frac{\overline{\gamma^2}}{l^2} \qquad (3)$$

which is the definition of $l(t)$. Now

$$\frac{d\overline{\gamma^2}}{dt} = -12D\,\frac{\overline{\gamma^2}}{l^2} \qquad (4)$$

still an indeterminate equation since it has two unknown functions.

The time history of $\overline{\gamma^2}$, and indeed of considerably more detailed functions, has been studied both theoretically and experimentally in the case of turbulence which is "decaying," i.e., dying out by viscous dissipation at the same time as it mixes the scalar contaminant. From a practical viewpoint it seems of interest to carry out a simple analysis under the postulate that the turbulent motion is statistically stationary in time.

In fact it is well known that isotropy and stationariness are contradictory specifications for a turbulent motion; an isotropic turbulence has a continuous viscous loss of kinetic energy to internal energy, with no balancing supply of energy from mean motion. A turbulence which receives energy from a mean flow field cannot be isotropic. Nevertheless, the postulate is used here primarily for the *small* structure of the fluctuation fields [for the derivatives, as in Equation (3)], and the requisite condition of "local isotropy" has been reasonably well confirmed by experiment (*2, 3*) since its original proposal by Kolmogoroff (*4*).

Before introduction of any postulate other than local isotropy, (4) may be schematically integrated:

$$\overline{\gamma^2}(t) = \overline{\gamma_0^2}\exp\left\{-12D\int_0^t \frac{dt_1}{l^2(t_1)}\right\} \qquad (5)$$

RESTRICTION TO STATIONARY TURBULENCE

Characteristic Reynolds and Péclet numbers of the fields are

$$R_\lambda \equiv \frac{1}{\sqrt{3}}\frac{q'\lambda}{\nu}\ ;\quad P_l \equiv \frac{1}{\sqrt{3}}\frac{q'l}{D} \qquad (6)^*$$

where q' is the root-mean-square resultant velocity fluctuation and λ is the dissipation scale of the turbulence, the Taylor microscale (*5*).

With sufficiently high R_λ and P_l and approximately equal large structures for the two fields, two different rough theoretical analyses (*1, 6, 7*) have predicted

$$\frac{l^2}{\lambda^2} \approx 2\,\frac{D}{\nu} \qquad (7)$$

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*The $\sqrt{3}$ factor is included to be consistent with the isotropic form $R_\lambda \equiv u'\lambda/\nu$.

An implied restriction in both theories is that the fluctuation decay rates are not sufficiently rapid to have influenced the form of power spectra (or correlation functions). In the present analysis, if it is now restricted to stationary turbulence, this suggests that (7) may not be valid for very large D/ν. In any case Equation (7) has received approximate experimental confirmation in the mixing of warm air behind a grid spanning a uniform wind-tunnel flow (*7*). It is hoped that these measurements will be done in water too.

With stationary turbulence, λ = constant, and so (7) gives l = constant and (5) reduces to

$$\overline{\gamma^2}(t) = \overline{\gamma_0^2} e^{-12(Dt/l^2)} \qquad (8)$$

The time constant of this exponential approach to uniformity is

$$\tau = \frac{l^2}{12D} \qquad (9)$$

When (7) applies, τ is expressible entirely in terms of the turbulence:

$$\tau = \frac{\lambda^2}{6\nu} \qquad (10)$$

APPLICATION TO A MIXER

In isotropic turbulence Taylor showed that the rate of dissipation of turbulent kinetic energy per unit mass is (*5*)

$$\Phi = 10\nu \frac{\overline{u^2}}{\lambda^2} \qquad (11)$$

For turbulence that is locally isotropic but not fully isotropic, it is prudent to replace (11) with

$$\Phi \approx \frac{10}{3} \nu \frac{\overline{q^2}}{\lambda^2} \qquad (12)$$

To maintain a steady state, the mixing device must supply energy to the fluid at this rate. If η is the efficiency of the mixing device (e.g., a paddle propeller), the total power input is

$$P = \frac{10}{3} \eta M \nu \frac{\overline{q^2}}{\lambda^2} \qquad (13)$$

where M is the mass of fluid.

In isotropic turbulence at large Reynolds number von Kármán and Howarth (*8*) deduced the approximate relation between dissipative scale λ and integral scale L

$$\frac{\lambda}{L} \sim \frac{1}{R_\lambda} \qquad (14)$$

In locally isotropic, stationary, shear turbulence this form can be inferred by equating the turbulent production and dissipation rates.

Introducing an empirical constant of proportionality gives

$$\frac{\lambda}{L} \approx \frac{A}{R_\lambda} \qquad (15)$$

or

$$q' \approx \frac{\sqrt{3} A L \nu}{\lambda^2} \qquad (15a)$$

With (13) and (15*a*), τ can be expressed in terms of quantities more directly related to the mixer L and P:

$$\tau \approx \left\{ \frac{\eta A^2}{21.6} \frac{ML^2}{P} \right\}^{1/3} \qquad (16)$$

$$\overline{\gamma^2}(t) \approx \overline{\gamma_0^2} e^{-t/\tau} \qquad (17)$$

From turbulence measurements in the approximately isotropic turbulence behind grids, (*10*)

$$A \approx 20$$

The mixer efficiency η will doubtless vary considerably with geometry and does not appear to have been measured in practical installations. Appropriate measurements for grid-generated turbulence in a wind tunnel would be the static pressure drop across the grid and the turbulent-energy–dissipation rate close behind. These also seem to be unavailable. But the efficiency of a fully developed pipe flow as turbulence generator is given by the data of Laufer (*9*). He finds that the mean-flow kinetic energy (generated by static pressure-gradient work) is "dissipated" about half to internal energy directly and half to turbulent kinetic energy (which also eventually becomes internal energy). Laufer's results appear to be a very insensitive function of Reynolds number, especially at large Reynolds number, and so this dependence is neglected in the present crude estimate. Thus, for want of more applicable empirical information, one takes

$$\eta \approx 0.5$$

so that

$$\overline{\gamma^2}(t) \approx \overline{\gamma_0^2} \cdot \exp \left\{ -3.3 \left(\frac{ML^2}{P} \right)^{1/3} \cdot t \right\} \qquad (18)$$

SCALING

Equation (16) provides simple scaling information for geometrically similar mixers at large Reynolds numbers.

An illustration is the determination of how much power is required to keep the same mixing time when the size is increased or decreased by a length factor K. The density is taken as constant. It has been found that the integral scale of turbulence is closely proportional to flow-system dimensions and independent of Reynolds number (*10*); hence

$$L' = KL \quad \text{and} \quad M' = K^3 M \qquad (19)$$

Therefore, according to Equation (16), $\tau' = \tau$ is achieved with

$$P' = K^5 P \qquad (20)$$

For $K > 1$ this considerable power requirement stems physically not only from the mass increase, but also from the need to increase q'. Equation (20) requires that both η and A be constant but is, of course, independent of the actual numerical values suggested in the previous section.

Equation (20) is not completely comparable with the results of direct dimensional reasoning on gross mixer properties by Kramers, Baars, and Knoll (*11*), because the angular speed of the stirrer appears explicitly. If, however, this is kept constant during scaling, the expression developed by them coincides with (20).

The combined dimensional reasoning and empirical approach of van de Vusse (*12*) apparently leads to $P' = K^{4.17}P$.

NOTATION

$\Gamma(x, y, z, t)$ = concentration
$\overline{\Gamma}$ = mean concentration, a constant when the field is statistically homogeneous in space
$\gamma(x, y, z, t) \equiv \Gamma - \overline{\Gamma}$ = concentration fluctuation
$\overline{\gamma^2}(t)$ = mean-square concentration fluctuation

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LITERATURE CITED

1. Corrsin, Stanley, *J. Aeronaut. Sci.*, **18**, No. 6 (June, 1951).
2. Townsend, A. A., *Proc. Cambridge Phil. Soc.*, **44**, 560 (1948).
3. Corrsin, Stanley, *J. Aeronaut. Sci.*, **16**, (December, 1949).
4. Kolmogoroff, A. N., *Comp. rend. acad. sci. U. R. S. S. (Doklady)*, **30**, 4 (1941).
5. Batchelor, G. K., "The Theory of Homogeneous Turbulence," Cambridge Univ. Press (1953).
6. Corrsin, Stanley, *J. Appl. Phys.*, **22**, 4 (1951).
7. Kistler, A. L., Vivian O'Brien, and Stanley Corrsin, *Natl. Advisory Comm., Aeronaut. Research. Memo* 54D19 (June, 1954).
8. von Kármán, Theodore, and Leslie Howarth, *Proc. Roy. Soc. (London)*, **A164**, 917 (January, 1938).
9. Laufer, John, *Natl. Advisory Comm. Aeronaut., Rep.* 1174 (1955).
10. Corrsin, Stanley, A.E. thesis, Calif. Inst. Technol. (1942).
11. Kramers, H., G. M. Baars, and W. H. Knoll, *Chem. Eng. Sci.*, **2**, 1 (1953).
12. van de Vusse, J. G., *ibid.*, **4**, Chap. 4 and 5, especially section 4.2 (1955).